SAINT IGNATIUS' COLLEGE

HSC Trial

2001

MATHEMATICS

9:00 – 12:05pm
Monday 3rd September 2001

Directions to Students

- Reading Time : 5 minutes
- Time Allowed : 3 hours
- Attempt ALL questions.
- Board approved calculators may be used.
- A standard integral table is provided
- Answer each question in a separate writing booklet and clearly label your name and teacher's name.

Total Marks 120

Attempt Questions 1 – 10

All questions are of equal value

Students are reminded that this is a trial examination only and cannot in any way guarantee the content or the format of the 2001 Mathematics Higher School Certificate examination

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Marks

Total marks (120)
Attempt Questions 1 - 10
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

QUESTION 1 (12 marks) Use a SEPARATE Writing Booklet.

(a) Find the value of e^{2x} when $x = 1.2$, correct to three decimal places. 2

(b) Solve the inequality $15 - 2x \leq 9$. 2

(c) Find the coordinates of the midpoint of the join of (-3, 6) and (1, 2). 2

(d) Write down the value of $\cos 150^\circ + \tan 240^\circ$ in exact form. 2

(e) Express $\dfrac{2}{\sqrt{6} - \sqrt{2}}$ with a rational denominator in simplest form. 2

(f) Solve $|2x - 3| = 9$. 2

QUESTION 2 (12 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) Differentiate the following functions:

(i) $(7-2x)^4$ **2**

(ii) xe^{3x} **2**

(iii) $\dfrac{x}{\tan x}$ **2**

(b) Find the following indefinite integrals:

(i) $\displaystyle\int \frac{1}{2x^2}\,dx$ **2**

(ii) $\displaystyle\int \frac{4x}{x^2-4}\,dx$ **2**

(c) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \sin 2x\,dx$ **2**

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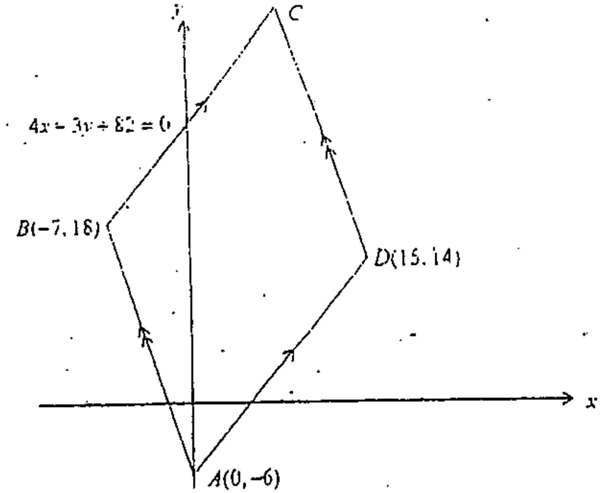
QUESTION 3 (12 marks) Use a SEPARATE Writing Booklet **Marks**



$ABCD$ is a parallelogram in which AB is parallel to DC and AD is parallel to BC.
A (0, -6), B (-7, 18), D (15, 14) are three of the vertices.
The equation of BC is $4x-3y+82=0$.

(a) Find the lengths of AB and AD. **2**

(b) Explain why $ABCD$ is a rhombus. **1**

(c) Find the equation of CD. **3**

(d) Show that the coordinates of C are (8, 38). **2**

(e) Show that the gradient of AC is $\frac{11}{2}$. **1**

(f) Find the gradient of BD. **1**

(g) Which geometrical property of a rhombus is illustrated by the results of (e) and (f)? **1**

(h) State another property of the diagonals of a rhombus (you do not need to prove this property). **1**

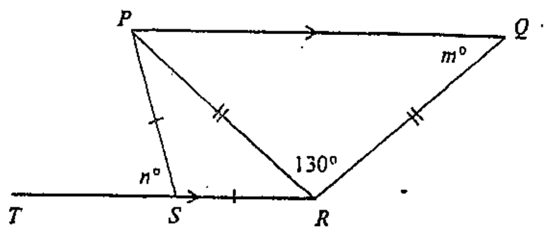
ESTION 4 (12 marks) Use a SEPARATE Writing Booklet. Marks

Simplify $\log_e(x^2-4)-\log_e(x-2)$. 2

Sketch the graph of the function $y = 1+\cos\frac{1}{2}x$ for $0 \le x \le 2\pi$. 2

In $\triangle PQR$, $PQ = 9$cm, $QR = 6$cm, and $\angle QPR = 25°$ 3

Show that there are two possible values for the size of $\angle QRP$, and find these values to the nearest degree.



In the diagram, PQ is parallel to TR, $PR = QR$, $PS = RS$; $\angle PRQ = 130°$.
T, S, and R are collinear.

(i) Draw a neat sketch of this diagram on your page. 1

(ii) $\angle PQR = m°$. Find the value of m giving reasons. 2

(iii) $\angle PST = n°$. Find the value of n giving reasons. 2

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Marks

QUESTION 5 (12 marks) Use a SEPARATE Writing Booklet

(a) 2

x	0	0.4	0.8	1.2
$f(x)$	2.6	3.4	4.8	4.4

The table lists the values of a function for four values of x.

Use the trapezoidal rule to estimate $\int_0^{1.2} f(x)\,dx$.

(b) The arc of the graph of $y = \sec x$ between $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$ is rotated about the x axis. 3
Find, in exact form, the volume of the solid formed.

(c) The graphs of $y = 5x - x^2$ and $y = x^2 - 3x$ intersect at the origin and at a point A.

(i) Show that the coordinates of A are (4, 4). 2

(ii) Draw a sketch of the two graphs on the same number plane. 1

(iii) Find the area enclosed by the two graphs 4

QUESTION 6 (12 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) In a bag there are six cubes which are identical except for their colour. Four cubes are blue and two cubes are red. **2**
Two cubes are withdrawn at the same time. What is the probability they are different colours ?

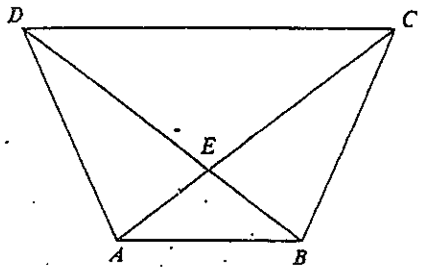
(b) In a large jar full of mint lollies, two-thirds of the lollies are green and one-third are white.
Three lollies are taken from the jar.
With the use of a tree diagram or otherwise, find the probability of taking:

(i) three green lollies **1**

(ii) one white and two green lollies **1**

(iii) at least one white lolly. **1**

(c)



In the diagram above, $ABCD$ is a quadrilateral in which $\angle DAB = \angle CBA$, $AD = BC$.
The diagonals AC and BD intersect at E.

(i) Copy this diagram onto your answer page and mark on it all the information. **1**

(ii) Prove ΔADB is congruent to ΔBCA. **3**

(iii) Explain why $AC = BD$. **1**

(iv) Prove that $AE = BE$. **1**

(v) Explain why $DE = EC$. **1**

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QUESTION 7 (12 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) Write $0.\dot{6}\dot{3} = 0.636363.......$ as the sum of terms of a geometric series. **3**
Hence find the value of $0.\dot{6}\dot{3}$ as a fraction in simplest form

(b) A man invested \$500 on 1st July each year, starting in 1996. Interest was at 6% per annum compounded annually.

(i) What is the value of his first deposit at 30th June 2010 ? **1**

(ii) What would be the total value of his investment at 30th June 2010 ? **3**

(c) 84 soft drink cans are stacked in rows, so that there is one less in each row than in the row immediately below. There are 15 cans in the bottom row.

(i) Show that the number of rows n is given by the equation **2**
$$n^2 - 31n + 168 = 0.$$

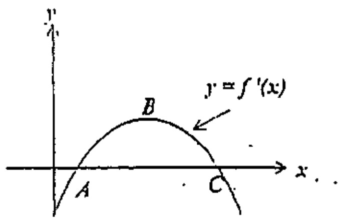
(ii) Find the number of rows of cans. **2**

(iii) How many cans are in the top row ? **1**

QUESTION 8 (12 marks) Use a SEPARATE Writing Booklet.

Marks

(a) 2



The diagram shows the graph of $y = f'(x)$, where $f'(x)$ is the derivative of $f(x)$.

What point, A, B, or C on this graph, corresponds to a maximum turning point on the graph of $y = f(x)$? Give reasons for your answer.

(b) For the quadratic equation $2x^2 - 6x + m = 0$, for what values of m are the roots real and different? 3

(c) The position of a particle, x cm from the origin, moving in a straight line, is given by

$$x = 6t^2 - t^3 \quad \text{for } t \geq 0, \text{ where } t \text{ is in seconds.}$$

(i) Find the initial position of the particle. 1

(ii) When does the particle return to its original position? 1

(iii) When does the particle come to rest? 2

(iv) What distance does the particle travel in the first 8 seconds? 3

QUESTION 9 (12 marks) Use a SEPARATE Writing Booklet.

Marks

(a) Using a graph of $y = \sin\theta$, or otherwise, solve the inequality: 2

$$\sin\theta \leq \frac{\sqrt{3}}{2} \quad \text{for } 0 \leq \theta \leq 2\pi.$$

(b) Consider the graph of the function $y = x - \log_e x$.

(i) Explain why the domain is $x > 0$. 1

(ii) Find the coordinates of the stationary point and determine its nature. 4

(iii) Explain why the curve is concave up for all x in the domain. 1

(iv) In how many points does the graph of $y = x - \log_e x$ cut the graph of $y = x$? Justify your answer. 2

(v) On the same set of axes, draw the graphs of $y = x$ and $y = x - \log_e x$. 2

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QUESTION 10 (12 marks) Use a SEPARATE Writing Booklet. Marks

(a) A sector of a circle has radius r and angle at the centre is θ radians. The area of the sector is 25 cm^2.

(i) Find an expression for θ in terms of r. 1

(ii) Show that the perimeter P cm of the sector is given by $P = \frac{50}{r} + 2r$. 1

(iii) Hence find the radius of the sector so that the perimeter is a minimum. 3

(iv) What is the minimum perimeter? 1

(b) Marketing studies show that if advertising for a particular product stops and other market conditions remain the same, then the sales of the unadvertised product will decline at a rate proportional to the current sales at any time.
That is: $\frac{dS}{dt} = -kS$ where S is the amount of sales at time t after advertising ceases.

(i) Show that $S = S_0 e^{-kt}$ satisfies this equation. 1

(ii) Explain why S_0 represents the amount of sales when advertising ceases. 1

A company agrees to start advertising again when sales have dropped to 75% of the sales when advertising ceased. Sales dropped to 95% of initial sales in two weeks.

(iii) Show that the value of k is 0.0256 to four decimal places. 2

(iv) When will the company start advertising again? 2

End of paper

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2 Unit – Question 1

(a) $e^{2x} = e^{2.4}$ when $x = 1.2$
$= 11.023$ (3 d p) [2]

(b) $15 - 2x \le 9$
$-2x \le -6$
$x \ge 3$ [2]

(c) $(-3, 6)\ (1, 2)$
Midpoint $= \left(\frac{-3+1}{2}, \frac{6+2}{2}\right)$
$= (-1, 4)$ [2]

(d) $\cos 150° + \tan 240° = -\cos 30° + \tan 60°$
$= -\frac{\sqrt{3}}{2} + \sqrt{3}$
$= \frac{\sqrt{3}}{2}$ [2]

(e) $\frac{2}{\sqrt{6}-\sqrt{2}} = \frac{2}{\sqrt{6}-\sqrt{2}} \times \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}$
$= \frac{2(\sqrt{6}+\sqrt{2})}{6-2}$
$= \frac{\sqrt{6}+\sqrt{2}}{2}$ [2]

(f) $|2x - 3| = 9$
$2x - 3 = 9$ or $2x - 3 = -9$
$2x = 12$ or $2x = -6$
$x = 6$ or $x = -3$ [2]

2 Unit - Question 2

(a) (i) $\frac{d}{dx}(7-2x)^4 = 4(7-2x)^3 \times (-2)$
$= -8(7-2x)^3$ [2]

(ii) $\frac{d}{dx}\, x e^{3x} = e^{3x} \times 1 + x \times 3e^{3x}$
$= e^{3x} + 3x\,e^{3x}$
or $e^{3x}(1+3x)$ [2]

(iii) $\frac{d}{dx}\frac{x}{\tan x} = \frac{\tan x \times 1 - x\sec^2 x}{\tan^2 x}$
$= \frac{\tan x - x\sec^2 x}{\tan^2 x}$ [2]

(b) (i) $\int \frac{1}{2x^2}\,dx = \int \frac{1}{2}x^{-2}\,dx$
$= \frac{1}{2} \times \frac{1}{-1} x^{-1} + C$
$= -\frac{1}{2x} + C$ [2]

(ii) $\int \frac{4x}{x^2-4}\,dx = 2\int \frac{2x}{x^2-4}\,dx$
$= 2\log_e(x^2-4) + C$ [2]

(c) $\int_0^{\frac{\pi}{3}} \sin 2x\,dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{3}}$
$= -\frac{1}{2}\left[\cos\frac{2\pi}{3} - \cos 0\right]$
$= -\frac{1}{2}\left[-\frac{1}{2} - 1\right]$
$= \frac{3}{4}$ [2]

3 Unit - Question 3

A(8, -6), B(-7, 18), D(15, 14).

(a) $AB = \sqrt{7^2+24^2} = 25$ units
$AD = \sqrt{15^2+20^2} = 25$ units [2]

(b) Opposite sides are parallel and a pair of adjacent sides are equal
OR it is a parallelogram with a pair of adjacent sides equal [1]

(c) Grad of $AB = -\frac{24}{7}$
Equation of CD $\quad y - 14 = -\frac{24}{7}(x-15)$
$7y - 98 = -24x + 360$
$24x + 7y - 458 = 0$ [3]

(d) Substitute $x = 8,\ y = 38$ into
CD $\quad 24 \times 8 + 7 \times 38 - 458 = 0$
BC : $\quad 4 \times 8 - 3 \times 38 + 82 = 0$
Since (8, 38) satisfies both equations, C is (8, 38)
OR $\quad 24x + 7y - 458 = 0 \qquad 24x + 7y = 458 \quad (1)$
$4x - 3y + 82 = 0 \qquad 24x - 18y = -492 \quad (2)$
$(1)-(2) \qquad 25y = 950$
$y = 38$
Subst $y = 38$ into $4x - 3y + 82 = 0$: $\quad 4x - 3 \times 38 + 82 = 0$
$4x = 12$
$x = 3$

Point C is (3, 38) [2]

(e) Grad of $AC = \frac{38-(-6)}{8-0} = \frac{44}{8} = \frac{11}{2}$ [1]

(f) Grad. of $BD = \frac{14-18}{15-(-7)} = \frac{-4}{22} = \frac{-2}{11}$ [1]

(g) Diagonals are perpendicular to each other $(m_1 m_2 = -1)$ [1]

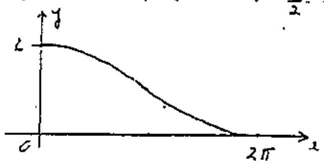
(h) The diagonals bisect each other. [1]

2 Unit - Question 4

(a) $\log_e(x^2-4) - \log_e(x-2) = \log_e \frac{(x-2)(x+2)}{x-2}$

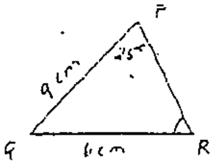
$= \log_e(x+2)$ [2]

(b) $y = 1 + \cos\frac{1}{2}x$ Period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$



[2]

(c)



$\frac{\sin R}{9} = \frac{\sin 25°}{6}$

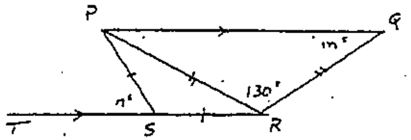
$\sin R = \frac{9 \times \sin 25°}{6}$

$= 0.6339$

$\angle R = 39°, 141°$

Since $141° + 25° < 180°$, $\angle PRQ = 39°, 141°$. [3]

(d) (i)



[1]

(ii) $\angle RPQ = \angle RQP$ (base angles of isosceles triangle)

$\therefore m + m + 130 = 180$ (angle sum of triangle)

$\therefore m = 25$ [2]

(iii) $\angle QPR = \angle PRS$ (alternate angles, $PQ \parallel TR$)

$\therefore \angle PRS = 25°$

$\therefore \angle SPR = 25°$ (base angles of isosceles triangle PSR)

$\therefore \angle PST = 25° + 25°$ (exterior angle of triangle)

$= 50°$

$\therefore n = 50$ [2]

2 Unit - Question 5

(a) $\int_1 \doteq \frac{0.5}{2}$ [illegible]

$\doteq 4.65$ [2]

(b) $V = \pi \int y^2\,dx$

$= \pi \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec^2 x\,dx$

$= \pi \left[\tan x\right]_{\pi/4}^{\pi/3}$

$= \pi \left[\tan\frac{\pi}{3} - \tan\frac{\pi}{4}\right]$

Volume $= \pi(\sqrt{3}-1)$ unit^3 [3]

(c) (i) $y = 5x - x^2$, $y = x^2 - 3x$

$5x - x^2 = x^2 - 3x$

$8x - 2x^2 = 0$

$2x(4-x) = 0$

$x = 0$ or $x = 4$.

When $x = 4$, $y = 20 - 16 = 4$

A is $(4, 4)$

OR Show that $(4,4)$ satisfies both equations

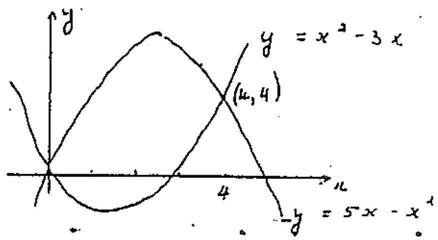
$y = 5x - x^2$. $y = 5 \times 4 - 4^2$

$= 4$

$y = x^2 - 3x$. $y = 4^2 - 4 \times 3$

$= 4$. [2]

(ii)



[1]

(iii) $A = \int_0^4 (5x - x^2) - (x^2 - 3x)\,dx$

$= \int_0^4 (8x - 2x^2)\,dx$

$= \left[4x^2 - \frac{2}{3}x^3\right]_0^4$

$= 64 - 42\frac{2}{3}$

$= 21\frac{1}{3}$.

Area $= 21\frac{1}{3}$ unit^2 [4]

2 Unit – Question 6

(a) 4 blue, 2 red

∴ P(different colours) $= P(B,R) + P(R,B)$

$= \frac{4}{6}\times\frac{2}{5} + \frac{2}{6}\times\frac{4}{5}$

$= \frac{8}{15}$ [2]

(b) $P(G) = \frac{2}{3}$, $P(W) = \frac{1}{3}$

(i) $P(3G) = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \frac{8}{27}$ [1]

(ii) $P(1W, 2G) = P(WGG) + P(GWG) + P(GGW)$

$= \frac{1}{3}\times\frac{2}{3}\times\frac{2}{3} + \frac{2}{3}\times\frac{1}{3}\times\frac{2}{3} + \frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}$

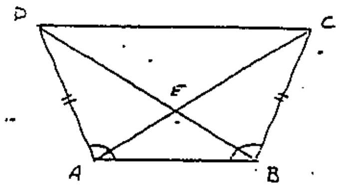
$= \frac{4}{9}$ [1]

(iii) P(at least one W) $= 1 - P(\text{no } W)$

$= 1 - \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}$

$= \frac{19}{27}$ [1]

(c) (i)



[1]

(ii) In △ADB, △BCA

AD = BC (given)

AB = AB (common)

∠DAB = ∠ABC (given)

∴ △ADB ≡ △BCA (SAS) [3]

(iii) AC = BD, corresponding sides in congruent triangles [1]

(iv) Because △ADB ≡ △BCA, ∠EAB = ∠EBA (corresp. angles in cong. △s)

∴ AE = BE (sides opposite equal angles) [1]

(v) Since BD = AC, BE = AE, then BD − BE = AC − AE

i.e. DE = EC [1]

2 Unit – Question 7

(a) (i) $0·63 + 0·0063 + 0·000063 + 0·00000063 + \dots$ OR $\frac{63}{100} + \frac{63}{10000} + \frac{63}{1000000} + \dots$

$= \frac{0·63}{1 - 0·01}$ $= \frac{\frac{63}{100}}{1 - \frac{1}{100}}$

$= \frac{63}{99}$

$= \frac{7}{11}$ $= \frac{7}{11}$ [3]

(b) (i) Value at 30 June $= \$500 \times 1·06^{14}$

$= \$1130.45$ [1]

(ii) Total value $= 500\times1·06^{14} + 500\times1·06^{13} + \dots + 500\times1·06$

$= \frac{500\times1·06\left[1·06^{14} - 1\right]}{1·06 - 1}$

$= \$11137.98$ [3]

(c) (i) A.P. is $15 + 14 + 13 + \dots$

For n rows, sum of n terms $= \frac{n}{2}\left[2\times15 + (n-1)\times(-1)\right] = 84$

$\frac{n}{2}(31 - n) = 84$

$n(31 - n) = 168$

$n^2 - 31n + 168 = 0$ [2]

(ii) $(n-7)(n-24) = 0$ OR $n = \frac{31 \pm \sqrt{31^2 - 4\times168}}{2}$

$n = 7$ or $n = 24$

∴ There are 7 rows

$= \frac{31 \pm 17}{2}$

$= 7, 24$ [2]

(iii) In row 7, no. of cans $= 15 + 6\times(-1)$

$= 9$ [1]

2 Unit - Question 8

(a) At a maximum turning point, the gradient changes from positive to negative ∴ C corresponds to a maximum turning point [2]

(b) $2x^2 - 6x + m = 0$

Roots are real and different if $\Delta > 0$

$36 - 4 \times 2 \times m > 0$

$36 > 8m$

$m < 4\frac{1}{2}$ [3]

(c) $x = 6t^2 - t^3, \quad t \geq 0$

(i) When $t = 0$, $x = 0$. Initial position at $x = 0$ [1]

(ii) When $x = 0$, $6t^2 - t^3 = 0$

$t^2(6 - t) = 0$

$t = 0, 6$

Returns to original position after 6 seconds [1]

(iii) $v = 12t - 3t^2$

When $v = 0$, $12t - 3t^2 = 0$

$3t(4 - t) = 0$

$t = 0, 4$

Comes to rest 4 seconds after starting [2]

(iv) When $t = 4$, $x = 6 \times 16 - 64$

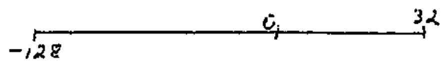
$= 32$

When $t = 8$, $x = 6 \times 64 - 8^3$

$= -128$

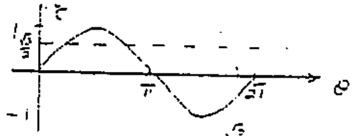
Distance travelled $= 32 + (32 + 128)$

$= 192$ cm [3]



2 Unit - Question 9

(a)



$\sin\theta \leq \frac{\sqrt{3}}{2}$

$0 \leq \theta \leq \frac{\pi}{3}, \quad \frac{2\pi}{3} \leq \theta \leq 2\pi$ [2]

(b) $y = x - \log_e x$

(i) Domain of $\log_e x$ is $x > 0$

domain of $y = x - \log_e x$ is $x > 0$ [1]

(ii) $\frac{dy}{dx} = 1 - \frac{1}{x}$

$= 0$ when $x = 1$

Stationary point at $(1, 1)$.

$\frac{d^2y}{dx^2} = \frac{1}{x^2} > 0$ at $x = 1$

∴ $(1,1)$ is a relative minimum [4]

(iii) $\frac{d^2y}{dx^2} = \frac{1}{x^2} > 0$ for all $x > 0$ since $x^2 > 0$ for $x > 0$.

∴ the curve is concave up for $x > 0$ [1]

(iv) Solve $y = x$, $y = x - \log_e x$ simultaneously

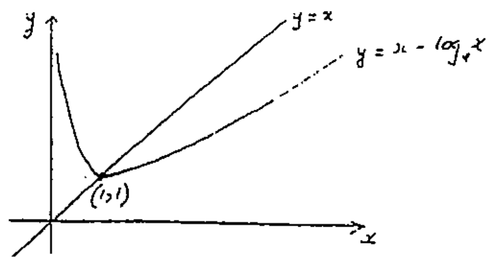
$x - \log_e x = x$

$-\log_e x = 0$

$x = 1$

The graphs cut at one point only [2]

(v)



[2]